Episteme: an online interdisciplinary, multidisciplinary & multi-cultural journal
Bharat College of Arts and Commerce, Badlapur, MMR, India
Volume 6, Issue 1 June 2017

MODIFIED Sl_0 *ALGORITHM*

By Shriniwas Acharya

Abstract

Now a days Mobile technology is widely used to communicate and various techniques are used to transfer the data. Signal to noise ratio is one of the main parameter used to improve the quality and accuracy of the signal. SISO technique used for Single input and Single output, for SIMO single input and Multiple Output with the same MIMO technology is also widely used with OFDM i.e. Orthogonal Frequency Division Multiplexing. The performance of the signal using MIMO-OFDM technique with Compressive Sensing and *Slo* Algorithm gives much better result in sparse channel estimation.

Keywords

MIMO, OFDM, Compressive Sensing Technique.

Introduction

Compressive sensing (CS) [18-19, 33] is a mathematical method to obtain a unique solution from an underdetermined linear system taking advantage of the prior knowledge that the true solution is sparse. Consider an underdetermined system of linear equations (USLE) with M equations and N unknowns ($M \times N$) as follows:

$$y = Ax \qquad (1)$$

Where the sensing matrix (with its columns being called atoms) $A \in \square^{M\times N}$, the unknown vector $x \in \square^{N\times 1}$and the measurement vector $y \in \square^{M\times 1}$, assuming that both A and y are exactly known. This problem is underdetermined and has infinite solutions. But if matrix A satisfies restricted isometry property (RIP) [20], and vector x is sparse (i.e. most elements of vector x are zero or close to zero), the unknown vector x should be imposed to arrive at a unique solution by various compressive sensing recovery algorithms. Thanks to the development of compressive sensing, many sparse channel estimation methods have been proposed for exploiting the channel sparsity. In this thesis, a new approach to estimating sparse multipath channels based on compressive sensing theory is proposed.

However, it is noteworthy that the channel impulse response (CIR) is simply modeled by a real-valued vector in most proposed papers, but as a matter of fact, the reference signals and the CIR are all complex-valued. Moreover, most conventional compressive sensing recovery methods, which require solving a ℓ_1 norm optimization problem or using greedy algorithm, are built on real-valued systems. Whereas for complex-valued systems, it is usually required to solve a second-order cone programming (SOCP) problem, which consumes much more computational cost and does not satisfy the requirement of CSI recovery speed [19, 22].

In general, the compressive sensing based sparse channel estimation method reduces the required training signals remarkably, while consumes more computational cost, than the conventional linear ones based on Nyquist paradigm. In 2005, Candes et al [26] proposed a theory of "compressive sampling" has emerged which shows that super-resolved signals or images can be recovered from far fewer data than what usually considered necessary. In 2009, Mohimani et al. [32] proposed smoothed ℓ_0 norm algorithm (SL0) for sparse representation. Other than Candes' most conventional compressive sensing recovery methods using ℓ_1 norm optimization or greedy algorithm, SL0 can easily solve the complex-valued USLE by minimizing a smoothed version of ℓ_0 norm directly, and was shown to be much faster and more accurate than the conventional algorithms. In view of the speed and accuracy of CSI recovery in complex sparse channel estimation, SL0 algorithm is more suitable than others. In [32], they proposed a complex sparse channel estimation method using SL0 algorithm. By applying the SL0 algorithm into the CIR recovery procedure, the sparse channel estimation not only can be achieved much faster and more accurate, but also works well in the complex-valued channel which is more reasonable in practice. In previous chapter, simulation results are provided to verify performance.

A comprehensive proof of the convergence of the SL0 algorithm exists for a specific set of parameters provided that an Asymmetric Restricted Isometry Property (ARIP) constraint is satisfied [32, 34]. The authors of the proof conclude that though theoretically satisfactory, the ARIP constraint leads to an unnecessarily pessimistic choice of parameter values. Motivated by this conclusion, we have carried out an extensive empirical analysis with the objective of finding the optimal parameter values. We suggest a modification to the SL0 algorithm which may greatly improve the overall performance.

In this thesis, we propose a complex sparse channel estimation method using modified SL0 algorithm. By applying the modified SL0 algorithm into the CIR recovery procedure, the sparse channel estimation not only can be achieved much faster and more accurate, but also works well in the complex-valued channel which is more reasonable in practice. Simulation results are provided to verify performance and complexity. The performance of sparse channel estimation is evaluated by mean-square-error (MSE), signal to noise ratio and bit-error rate (BER), while computational complexity is measured roughly by CPU time of the computer. Figure shows the complex valued sparse signal using compressive sensing.

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Volume 6, Issue 1 **June 2017**

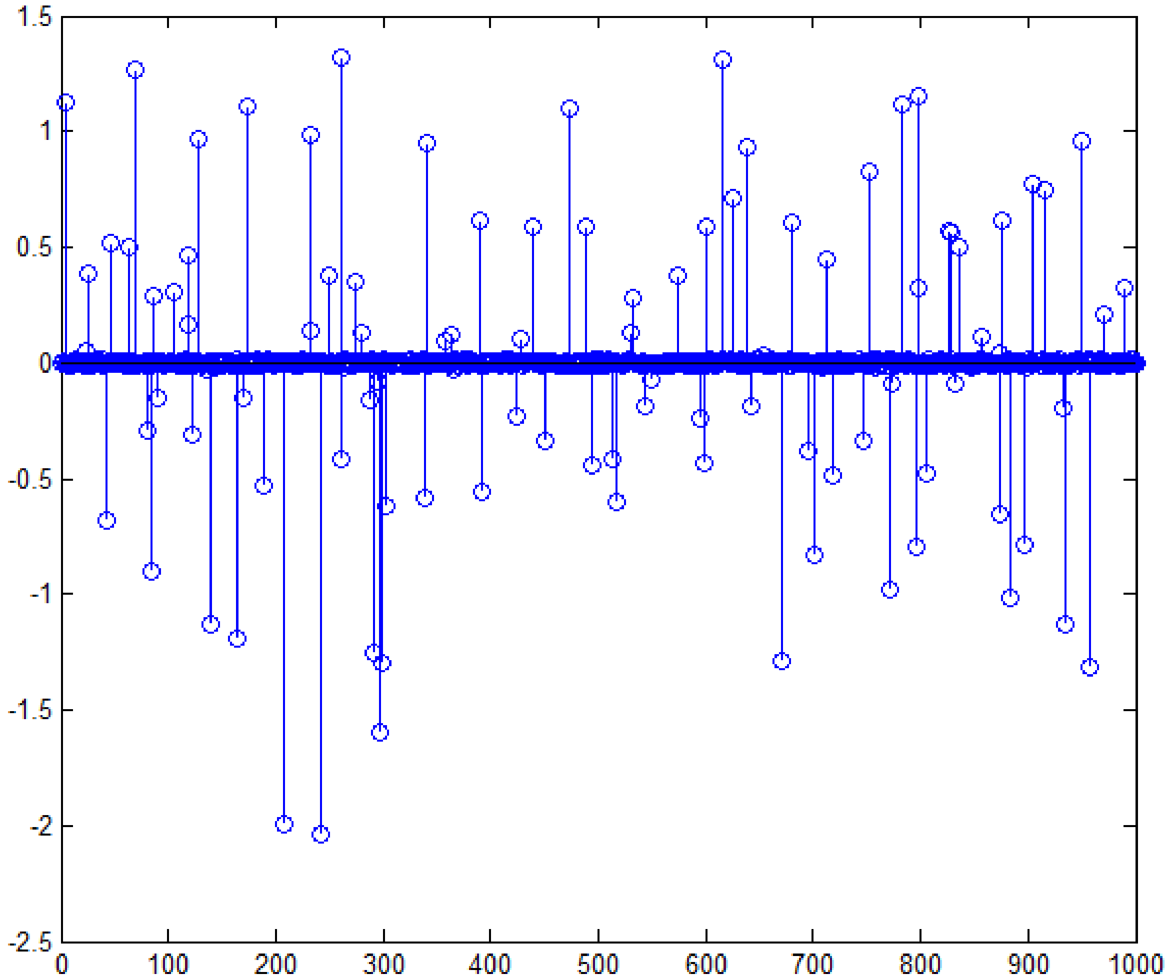

Figure: Complex valued sparse signal

Proposed Algorithm

To achieve this, an intuitional method is to directly seek for solutions to (P_0): $min\|x\|_0$ $subject\ to\ Ax = y$ however, directly minimizing the ℓ_0 norm, a discontinuous function of that vector, is intractable as it requires an exhausting combinatorial search. Moreover, since any small amount of noise completely changes the ℓ_0 norm of a vector, this method is not robust in noisy settings. But modified SL0 algorithm, deals with the P_0 problem in a different manner. Modified SL0 approximates ℓ_0 norm of vector $x$with a continuous function of$m - F_\sigma(x)$. The parameter σ may be used to control the accuracy with which $f_\sigma$approximates the kronecker delta function as $\sigma \to 0$. Therefore, instead of minimizing $\|x\|_0$ subject to data, modified SL0 attempts to solve the problem $Q: max_x F_\sigma(x)$subject to $y = Ax$. However, it is observed thatF_σ, for small σ, contains numerous local maxima, while having no local maxima for sufficiently large values of σ. Therefore, to avoid getting trapped into local maxima, modified SL0 solves a sequence of problems of the form $Q_\sigma: max_x F_\sigma(x)$subject to $y = Ax$, decreasing σat each step, and initializing the next step at the maximum of the previous larger value of σ. Each $Q_\sigma$is solved using few iterations of gradient ascent. Thus, modified SL0 algorithm consists of two loops: the external loop that is responsible for gradually decreasing of $\sigma$and the internal loop, being a simple steepest ascent algorithm for finding the maximum of $Q_\sigma$for given σ. The final modified SL0 algorithm, which is obtained by applying the main idea of above on the Gaussian family, for extracting the CIR vector $x$from the received pilot subcarriers Y_P is given in Fig. 2. The convergence analysis of modified SL0 has been thoroughly considered in [32] and it was shown that, under mild conditions, the sequence of maximum of $Q_\sigma$indeed converges to the unique minimum of P_0, whenever such answer exists. Moreover, modified SL0, solving the P_0 problem directly hence no need to differentiate real-valued or complex-valued data, runs significantly faster than the competing algorithms, while producing answers with the same or better accuracy [32].

By introducing an update of the L parameter, the phase transition of the modified SL0 algorithm can be improved. But computational time to get improved result is comparatively more. Hence to avoid this drawback, we keep same L parameter as that of original SL0 algorithm. By carefully selecting sequences of μ's and σ's, the phase transition of the modified SL0 algorithm can be even further improved to consistently lie on or above that of the l_1 approach. In brief, we chose the step-size in the order of 10^{-3} for the first few σ's and in the order of 1 for the last σ's. Also, we chose the initial σ based on knowledge of the indeterminacy δ to improve the phase transition across all δ.

Based on our experimental results, we propose the following strategy. We choose a sequence of step-size of$\mu = (1.8, 0.7, 0.4)$. Furthermore, an inversely proportional relation between δ and the initial value of σ yielded the most promising phase transition. Specifically, we choose an initial $\sigma = 1 * \max|\hat{x}|$. Finally, a gradually increasing L for decreasing σ still provides an improvement for the updated parameter choices. Generally, this measure has proved to be a good indicator of convergence and significantly reduced the average number of iterations taken in the inner loop. We now propose the modified smoothed *l*0 norm algorithm with improved parameter selection.

Modified SL0 Algorithm

1. Initialize: $\sigma_{up} = 0.5, \sigma_{min} = 0.001,$
2. $\mu = [1.8, 0.7, 0.4],$
3. $L = 3$
4. $\hat{x} \rightarrow A^{\dagger}y$
5. $\sigma \leftarrow 1 * \max|\hat{x}|$
6. While $\sigma > \sigma_{min}$ do
7. for $i = 1 \ldots. L$ do
8. $\delta \leftarrow \hat{x} \circ \exp\left[-\frac{\hat{x}\circ\hat{x}}{0.5*\sigma^2}\right]$
9. $\hat{x} \leftarrow \hat{x} - \mu\delta$
10. $\hat{x} \leftarrow \hat{x} - A^{\dagger}(A\hat{x} - y)$
11. $\sigma \leftarrow \sigma.\sigma_{up}$

Note that the algorithm has two loops: the external loop which corresponds to the basic ideas for finding the sparsest solution, and the internal loop which is a simple steepest ascent algorithm for maximizing $F_{\sigma}(x)$for a fixed σ. In the analysis of this section, it is assumed that the maximization of $F_{\sigma}(x)$has been exactly done for a fixed σ (the maximization algorithm has not got trapped into local maxima). Note that we had proposed the gradual decrease in σ to escape from getting trapped into local maxima when maximizing $F_{\sigma}$for a fixed σ. A theoretical study to find the series σ_j, j = 1,…J, which guaranties the convergence is very tricky (if possible) and is not considered in this thesis.

Conclusion

In the sparse channel estimation, various algorithms and various techniques are used. Compressive sense technique with *Sl0* Algorithms gives better performance and Signal to Noise ratio vs Mean Square Error and Bit Error Rate is improved.

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Bio

Mr. Acharya Shriniwas is an Asst. Professor in Department of Computer Science and Information Technology, Bharat College. He can be contacted at me.electronicsengineer@gmail.com