

APPLICATIONS OF MATRICES IN DIGITAL IMAGE

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Abstract

The images you see on internet pages and the photos you take with your mobile phone are examples of digital images. It is possible to represent this kind of image using matrices. Objective of this paper is to present the application of matrices i.e. processing of digital Images for teachers & students. The subject is vast, rich, and modern. Note that the mathematical tools related to this topic go far beyond matrices.

Keywords

Matrix, Pixel, Raster image, Vector image, Arithmetic mean, Decomposition of matrix.

Introduction

Digital Anne's life has changed considerably due to implementation of these digital technologies. Everything is simpler, more streamlined and 'user friendly'. The paperwork has been eliminated. Robots are present in almost every part of our life, at home, at work, in the shops. We use digital technologies in day to day life such as a smart phone, email, ipad and laptop. Digital Images is one of them. A matrix has huge numbers of applications including graph theory, computer graphics, cryptography, solving linear equations etc. Digital image processing is one of them. Different operations on matrix elements like transpose of matrix, standard arithmetic mean of the component, multiplication by a scalar, sum of matrices, decomposition of matrix etc. affect the corresponding image.

Objective of the paper

1. To understand the applications of Matrix.
2. To understand the meaning of digital images & types digital images.
3. To correlate relation between application of Mathematics (Matrices) & Digital world (digital images).

Matrix

The arrangement of rows & columns in specific manner is called matrix. The entries may be a number, an expression, a symbol

$$\begin{matrix} 2 & 3 & 3 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{matrix}$$

The dimension of the above matrix is 3×3 . Because it contains 3 rows & 3 columns

In mathematics, a **matrix** (plural **matrices**) is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

Digital Image

The images we can see on internet pages or images taken by mobile are the example of digital images. There are two types of digital images (Depending on whether the resolution is fixed or not)

1. Vector Image – These images are constructed using mathematical formulas rather than individual colored blocks, vector file types such as EPS, AI & PDF. These images are excellent for creating graphics that frequently require resizing. For example Company logo, brand graphics. Vector images are alternatively allow more flexibility as compared to other types. `

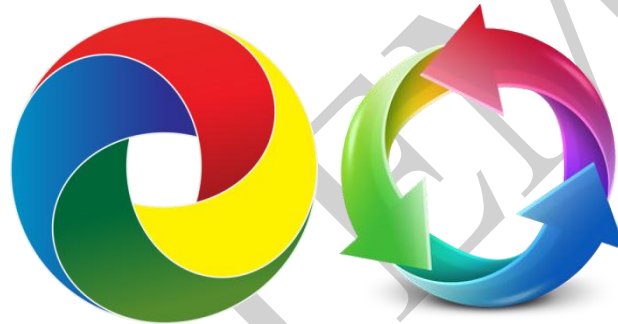


Figure 1: Vector Images

2. Raster Image/Bitmaps -Raster images have a finite set of digital values, called picture elements or pixel. Raster images are stored in image files with varying formats. In computer graphics it is also called a dot matrix data structure, representing a generally rectangular grid of pixels, or points of color, viewable via a monitor, paper, or other display medium.

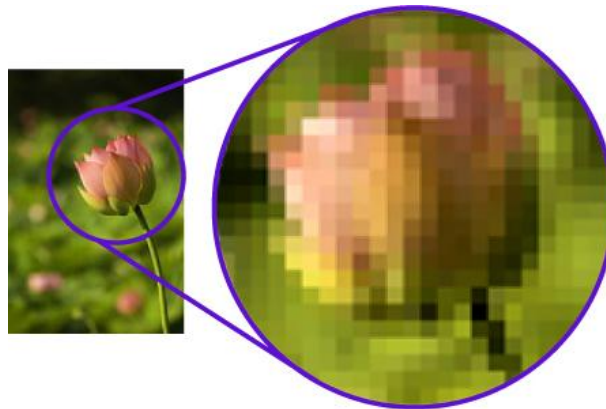


Figure 2: Raster Image

Pixel- Pixels are the smallest individual element in a raster image, which can hold one color at a time. The intensity of pixel is variable. In color imaging systems, a color is typically represented by three or four component intensities such as red, green, and blue, or cyan, magenta, yellow, and black.

Binary Image- The digital image using only two colors are called binary or Boolean image. Typically, the two colors used for a binary image are black and white, In that 0 is used for black color & 1 is used for white color. A binary image can be stored in memory as a bitmap, a packed array of bits. A very important characteristic of a binary image is the distance transform. This gives the distance of every set pixel from the nearest unset pixel.

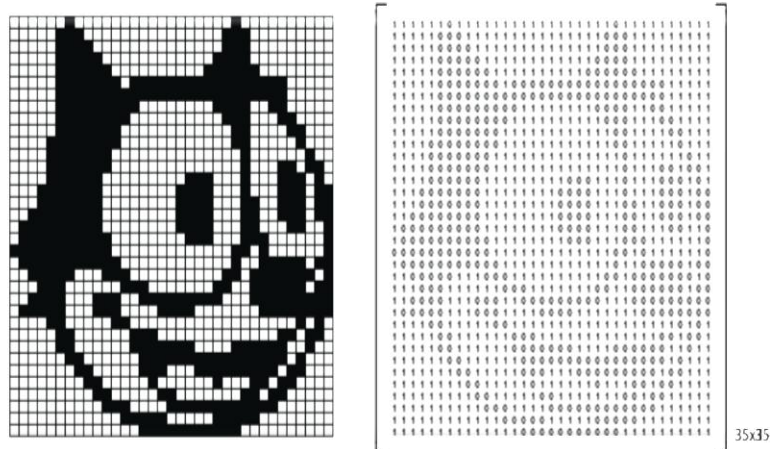


Figure 3: Binary Image

Color Image-A (digital) color image is a digital image that includes color information for each pixel. For visually acceptable results, it is necessary (and almost sufficient) to provide three samples (*color channels*) for each pixel, which are interpreted as coordinates in some color space. The RGB color space is commonly used in computer displays, but other spaces such as YCbCr/YCC, HSV (hue, saturation and lightness) and are often used in other contexts. A color image in RGB system has three values (or channels) per pixel and they measure the intensity and chrominance of light. The actual information stored in the digital image data is the brightness information in each spectral band.

A color image is usually stored in memory as a raster map, a two-dimensional array of small integer triplets; or (rarely) as three separate raster maps, one for each channel.

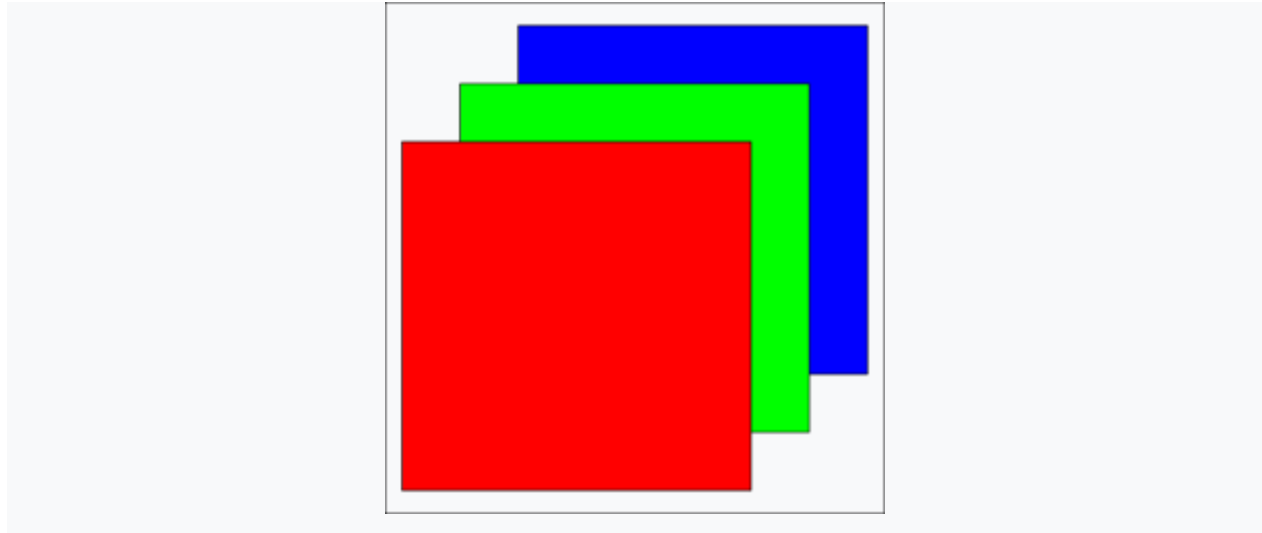


Figure 4: Separate R, G, and B image layers

Eight bits per sample (24 bits per pixel) seem adequate for most uses, but faint banding artifacts may still be visible in some smoothly varying images, especially those subject to processing. Particularly demanding applications may use 10 bits per sample or more.

On the other hand, some widely used image file formats and graphics cards may use only 8 bits per pixel, i.e., only 256 different colors, or 2–3 bits per channel. Converting continuous-tone images like photographs to such formats requires dithering and yields rather grainy and fuzzy results.

Graphics cards that support 16 bits per pixel provide 65536 distinct colors, or 5–6 bits per channel. This resolution seems satisfactory for non-professional uses, even without dithering.

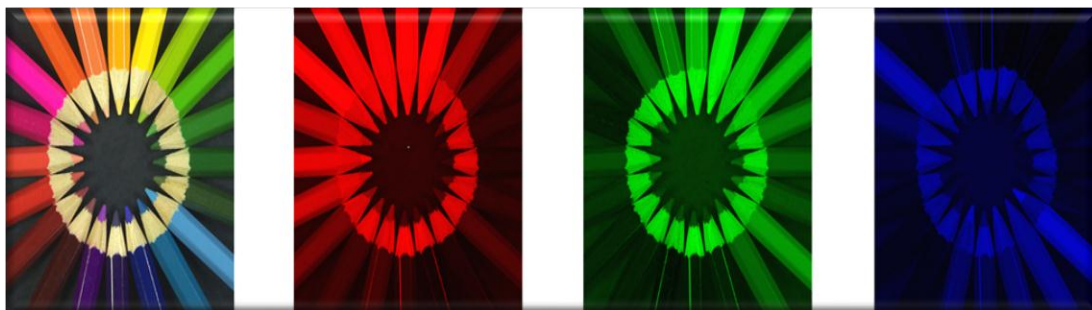


Figure 5: Color Image

Digital image processing and operations with matrices

1. **Transpose of matrix:** The transpose of a matrix is a new matrix whose rows are the columns of the original.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Transpose of a matrix

Consider the following digital image $A = (a_{i,j})$, then image $B = (b_{i,j})$ is the transpose of matrix A. That is

$$B = (b_{i,j}) = (a_{j,i}) = A^T,$$

So, Matrix transpose is used in changing the angle (Rotation) of image.



2. **Multiplication by a scalar & Sum of Matrices:** Using the operations of Matrix multiplication by a scalar and sum of matrices, it is possible to create an image transition effects. For example transitions used in Power Point Presentation, Slide shows etc.

If we consider two gray scale images of the same size, represented by the matrices A & Z. For each scalar (real number) r in the interval $[0, 2]$, we define the matrix

$$M(r) = (1-r)A + rZ$$

Note that $M(0) = A$, $M(2) = Z$ and for each r between 0 & 2, the elements of the matrix $M(r)$ are between the elements of the matrices A to Z.

Therefore, If r varies from 0 to 2, the matrix $M(r)$ varies from A to Z.

3. **Standard Arithmetic Mean (SAM):** SAM is defined the average of a set of numerical values calculated by adding them together and dividing by the number of terms in the set.

$$\text{That is, } c = \frac{a+b}{2} \text{ (Arithmetic mean of two numbers)}$$

If we take arithmetic mean of component matrices R, G and B from the original color image, we will get gray scale version of original image.



Figure 6: Original Color Image

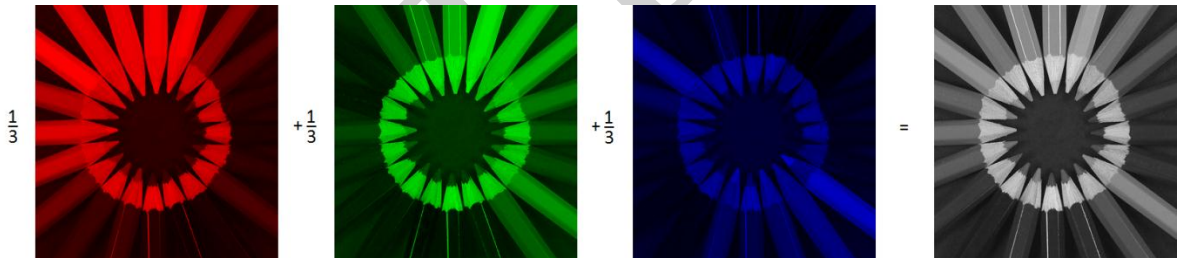


Figure 7: Arithmetic mean of the component matrices

4. **Singular Value Decomposition:** is a factorization of real or complex matrix. Consider, therefore, a *singular value decomposition* (SVD), that consists of writing a matrix $A_{m \times n}$ as the product of three matrices:

$$A_{m \times n} = U_{m \times m} S_{m \times n} V_{n \times n}^T,$$

Where U & V are orthogonal matrices and S is matrix whose elements s_{ij} are equal to zero for $i \neq j$ for example

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = USV^T$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \end{bmatrix}^T.$$

Observe that if $u_1, u_2, u_3, \dots, u_m$ are the columns of the matrix U and $v_1, v_2, v_3, \dots, v_n$ are the columns of the matrix V , then

$$A = USV^T = s_{1,1}u_1v_1^T + s_{2,2}u_2v_2^T + \dots + s_{k,k}u_kv_k^T.$$

Suppose that a gray scale image of size 1000×1000 must be transmitted from a satellite to a laboratory on Earth. For this operation satellite would have to send 1 million numbers (one for each pixel). But if use decomposition of matrices we need to send first elements $s_{i,j}$ of the matrix S of the SVD decomposition for A . Then the satellite sends the 20 first columns of two matrices i.e. $20 \cdot 1000 + 20 \cdot 1000 + 20 = 40020$ Upon receiving these data, the laboratory on Earth calculates the matrix

$$s_{1,1}u_1v_1^T + s_{2,2}u_2v_2^T + \dots + s_{20,20}u_{20}v_{20}^T$$

That will give an approximation of the original image.

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Bio

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